UNIT 17 CHI-SQUARE TEST

STRUCTURE

17.0 OBJECTIVES

After studying this unit, you should be able to:

- explain and interpret interaction among attributes,
- use the chi-square distribution to see if two classifications of the same data are independent of each other,
- use the chi-square statistic in developing and conducting tests of goodness-of-fit, and
- analyse the independence of attributes by using the chi-square test.

17.1 CHI-SQUARE DISTRIBUTION

In the previous two units, you have studied the procedure of testing hypothesis and using some of the tests like Z-test and t-test. In one sample test you have learned tests to determine whether a sample mean or proportion was significantly different from the respective population mean or proportion. But in practice the requirement in your research may not be confined to only testing of one mean/proportion of a population. As a researcher you may be interested in dealing with more than two populations. For example, you may be interested in knowing the differences in consumer preferences of a new product among people in the north, the south, and the north-east of India. In such situations the tests you have learned in the previous units do not apply. Instead you have to use chi-square test.

Chi-square tests enable us to test whether more than two population proportions are equal. Also, if we classify a consumer population into several categories (say high/medium/low income groups and strongly prefer/moderately prefer/indifferent/do not prefer a product) with respect to two attributes (say consumer income and consumer product preference), we can then use chi-square test to test whether two attributes are independent of each other. In this unit you will learn the chi-square test, its applications and the conditions under which the chi-square test is applicable.

17.2 CHI-SQUARE DISTRIBUTION

The chi-square distribution is a probability distribution. Under some proper conditions the chi-square distribution can be used as a sampling distribution of chi-square. You will learn about these conditions in section 17.5 of this unit. The chi-square distribution is known by its only parameter – number of degrees of freedom. The meaning of degrees of freedom is the same as the one you have used in student t-distribution. Figure 17.1 shows the three different chi-square distributions for three different degrees of freedom.

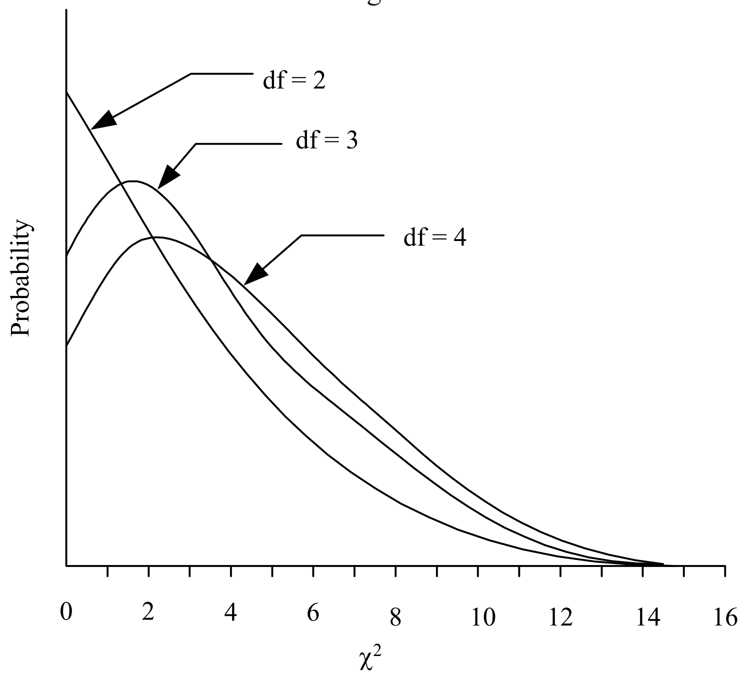

Figure 17.1. Chi-Square Sampling Distributions for df=2, 3 and 4

It is to be noted that as the degrees of freedom are very small, the chi-square distribution is heavily skewed to the right. As the number of degrees of freedom increases, the curve rapidly approaches symmetric distribution. You may be aware that when the distribution is symmetric, it can be approximated by normal distribution. Therefore, when the degrees of freedom increase sufficiently, the chi-square distribution approximates the normal distribution. This is illustrated in Figure 17.2.

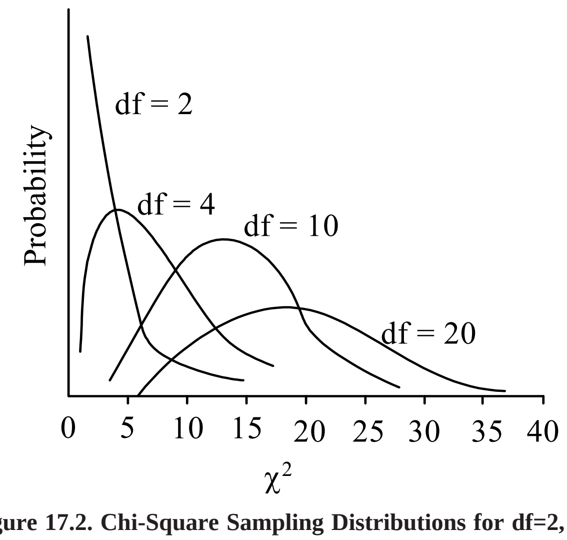

Figure 17.2. Chi-Square Sampling Distributions for df=2, 4, 10, and 20

Like student t-distribution there is a separate chi-square distribution for each number of degrees of freedom. Appendix Table-1 gives the most commonly used tail areas that are used in tests of hypothesis using chi-square distribution. It will explain how to use this table to test the hypothesis when we deal with examples in the subsequent sections of this unit.

17.3 CHI-SQUARE TEST FOR INDEPENDENCE OF ATTRIBUTES

Many times, the researchers may like to know whether the differences they observe among several sample proportions are significant or only due to chance. Suppose a sales manager wants to know consumer preferences of consumers who are located in different geographic regions of a country, of a particular brand of a product. In case the manager finds that the difference in product preference among the people located in different regions is significant, he/she may like to change the brand name according to the consumer preferences. But if the difference is not significant then the manager may conclude that the difference, if any, is only due to chance and may decide to sell the product with the same name. Therefore, we are trying to determine whether the two attributes (geographical region and the brand name) are independent or dependent. It should be noted that the chi-square test only tells us whether two principles of classification are significantly related or not, but not a measure of the degree or form of relationship. We will discuss the procedure of testing the independence of attributes with illustrations. Study them carefully to understand the concept of χ^2 test.

Illustration 1

Suppose in our example of consumer preference explained above, we divide India into 6 geographical regions (south, north, east, west, central and north east). We also have two brands of a product brand A and brand B.

The survey results can be classified according to the region and brand preference as shown in the following table.

Region	Consumer preference		Total
	Brand A	Brand B	
South	64	16	80
North	24	6	30
East	23	7	30
West	56	44	100
Central	12	18	30
North-east	12	18	30
Total	191	109	300

In the above table the attribute on consumer preference is represented by a column for each brand of the product. Similarly, the attribute of region is represented by a row for each region. The value in each cell represents the responses of the consumers located in a particular region and their preference for a particular brand. These cell numbers are referred to as observed (actual) frequencies. The arrangement of data according to the attributes in cells is called a **contingency table**. We describe the dimensions of a contingency table by first stating the number of rows and then the number of columns. The table stated above showing geographical region in rows (6) and brand preference in columns (2) is a 6 × 2 contingency table.

In the 6 × 2 contingency table stated above (the example of brand preference) each cell value represents a frequency of consumers classified as having the corresponding attributes.We also stated that these cell values are referred to as

observed frequencies. Using this data we have to determine whether or not the consumer geographical location (region) matters for brand preference. Here the null hypothesis (H_0) is that the brand preference is not related to the geographical region. In other words, the null hypothesis is that the two attributes, namely, brand preference and geographical location of the consumer are independent. As a basis of comparison, we use the sample results that would be obtained on the average if the null hypothesis of independence was true. These hypothetical data are referred to as the **expected frequencies**.

We use the following formula for calculation of expected frequencies (E).

$$E = \frac{\text{Row total} \times \text{Column total}}{\text{Total}}$$

For example, the cell entry in row-1 and column-2 of the brand preference 6x2 contingency table referred to earlier is:

$$E = \frac{80 \times 191}{300} = \frac{15280}{300} = 50.93$$

Accordingly, the following table gives the calculated expected frequencies for the rest of the cells of the 6x2 contingency table.

Calculation of the Expected Frequencies

Region	Consumer Preference		
	Brand A	Brand B	Total
South	(80×191)/300 = 50.93	(80×109)/300 = 29.07	80
North	(30×191)/300 = 19.10	(30×109)/300 = 10.90	30
East	(30×191)/300 = 19.10	(30×109)/300 = 10.90	30
West	(100×191)300= 63.67	(100×109)/300 =36.33	100
Central	(30×191)300 = 19.10	(30×109)/300 = 10.90	30
Northern	(30×191)/300 = 19.10	(30×109)/300 = 10.90	30
Total	191	109	300

We use the following formula for calculating the chi-square value.

$$\chi^2 = \Sigma \frac{(O_i - E_i)}{E_i}$$

Where, χ^2 = chi-square; O_i = observed frequency; E_i = expected frequency; and

Σ = sum of.

To ascertain the value of chi-square, the following steps are followed.

1) Subtract E_i from O_i for each of the 12 cells and square each of these differences $(O_i–E_i)^2$.

2) Divide each squared difference by E_i and obtain the total, i.e., $\Sigma \frac{(O_i - E_i)^2}{E_i}$.

This gives the value of chi-squares which may be ranged from zero to infinity. Thus, value of χ^2 is always positive.

Now we rearrange the data given in the above two tables for comparing the observed and expected frequencies. The rearranged observed frequencies, expected frequencies and the calculated χ^2 value are given in the following Table.

Row/Column	Observed frequencies (O_i)	Expected frequencies (E_i)	$(O_i–E_i)$	$(O_i–E_i)^2$	$(O_i–E_i)^2/E_i$
(1,1)	64	50.93	13.07	170.74	3.35
(2,1)	24	19.10	4.90	24.01	1.26
(3,1)	23	19.10	3.90	15.21	0.80
(4,1)	56	63.67	–7.67	58.78	0.92
(5,1)	12	19.10	–7.10	50.41	2.64
(6,1)	12	19.10	–7.10	50.41	2.64
(1,2)	16	29.07	–13.07	170.74	5.87
(2,2)	6	10.90	–4.90	24.01	2.20
(3,2)	7	10.90	–3.90	15.21	1.40
(4,2)	44	36.33	7.67	58.78	1.62
(5,2)	18	10.90	7.10	50.41	4.62
(6,2)	18	10.90	7.10	50.41	4.62
	300	300			χ^2 = 31.94

With r × c (i.e. r-rows and c-columns) contingency table, the degrees of freedom are found by (r–1) x (c–1). In our example, we have 6 × 2 contingency table. Therefore, we have (6–1) × (2–1) = 5 × 1 = 5 degrees of freedom. Suppose we take 0.05 as the significance level (a). Then at 5 degrees of freedom and a = 0.05 significance level the table value (from Appendix Table-4) is 11.071. Since the calculated χ^2 value (31.94) is greater than the table value of (11.071), we reject the null hypothesis and conclude that the brand preference is not independent of the geographical location of the customer. Therefore, the sales manager needs to change the brand name across the regions.

Illustration 2

A TV channel programme manager wants to know whether there are any significant differences among male and female viewers between the type of the programmes they watch. A survey conducted for the purpose gives the following results.

Type of TV programme	Viewers Sex		
	Male	Female	Total
News	30	10	40
Serials	20	40	60
Total	50	50	100

Calculate χ^2 statistic and determine whether type of TV programme is independent of the viewers' sex. Take 0.10 significance level.

Solution: In this example, the null and alternate hypotheses are:

H_0: The viewers sex is independent of the type of TV programme (There is no association among the male and female viewers).

H_1: The viewers sex is not independent of the type of TV programme.

We are given the observed frequencies in the problem. The expected frequencies are calculated in the same way as we have explained in illustration 1. The following table gives the calculated expected frequencies.

Type of TV Programme	Viewers Sex		
	Male	Female	Total
News	(40×50)/100 = 20	(40×50)/100 = 20	40
Serials	(60×50)/100 = 30	(60×50)/100 = 30	60
Total	50	50	100

Now we rearrange the data on observed and expected frequencies and calculate the χ^2 value. The following table gives the calculated χ^2 value.

(Row, Column)	Observed frequencies (O_i)	Expected frequencies (E_i)	$(O_i–E_i)$	$(O_i–E_i)^2$	$(O_i–E_i)^2/E_i$
(1,1)	30	20	10	100	5.00
(2,1)	20	30	–10	100	3.33
(1,2)	10	20	–10	100	5.00
(2,2)	40	30	10	100	3.33
					χ^2 =16.66

Since we have a 2x2 contingency table, the degrees of freedom will be (r–1) × (c–1) = (2–1) × (2–1) = 1× 1 = 1. At 1 degree of freedom and 0.10 significance level the table value (from Appendix Table-4) is 2.706. Since the calculated χ^2 value (16.66) is greater than table value of χ^2 (2.706) we reject the null hypothesis and conclude that the type of TV programme is dependent on viewers' sex. It should, therefore, be noted that the value of χ^2 is greater than the table value of x^2 the difference between the theory and observation is significant.

Self Assessment Exercise A

1) The following are the independent testing situations, calculated chi-square values and the significance levels. (i) state the null hypothesis, (ii) determine the number of degrees of freedom, (iii) calculate the corresponding table value, and (iv) state whether you accept or reject the null hypothesis.

   a) Type of the car (small, family, luxury) versus attitude by sex (preferred, not preferred). $\chi^2 = 10.25$ and a = 0.05.

   b) Income distribution per month (below Rs 10000, Rs 10000-20000, Rs 20000-30000, Rs 30000 and above) and preference for type of house with number of bed rooms (1, 2, 3, 4 and above). $\chi^2 = 28.50$ and a = 0.01.

   c) Attitude towards going to a movie or for shopping versus sex (male, female). $\chi^2 = 8.50$ and a = 0.01.

   d) Educational level (illiterate, literate, high school, graduate) versus political affiliation (CPI, Congress, BJP, BSP). $\chi^2 = 12.65$ and $\alpha = 0.10$.

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2) The following are the number of columns and rows of a contingency table. Determine the number of degrees of freedom that the chi-square will have

   a) 6 rows, 6 columns   c) 3 rows, 5 columns

   b) 7 rows, 2 columns   d) 4 rows, 8 columns

   ....

3) A company has introduced a new brand product. The marketing manager wants to know whether the preference for the brand is distributed independent of the consumer's education level. The survey of a sample of 400 consumers gave the following results.

	Illiterates	Literates	High School	Graduate	Total
Bought new brand	50	55	45	60	210
Did not buy new brand	50	45	55	40	190
Total	100	100	100	100	400

   a) Calculate the expected frequencies and the chi-square value.

   b) State the null hypothesis.

   c) State whether you accept or reject the null hypothesis at a = 0.05.

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17.4 CHI-SQUARE TEST FOR GOODNESS OF FIT

In unit 14, you have studied some probability distributions such as binomial, Poisson and normal distributions. When we consider a sample data from a population we try to assume the type of distribution the sample data follows. The chi-square test is useful in deciding whether a particular probability distribution such as the binomial, Poisson or normal distribution is the appropriate probability distribution. This allows us to validate our assumption about the probability distribution of the sample data. The chi-square test procedure used for this purpose is called goodness-of-fit test. The test also indicates whether or not the frequency distribution for the sample population has a particular shape, such as the normal curve (symmetric distribution). This can be done by testing whether there is a significant difference between an observed frequency distribution and an assumed theoretical frequency distribution. Thus by applying chi-square test for goodness of fit, we can determine whether the observed data constitutes a sample drawn from the population with assumed theoretical distribution. In this section we use chi-square test for goodness-of-fit to make inferences about the type of distribution.

The logic inherent in the chi-square test allows us to compare the observed frequencies (O_i) with the expected frequencies (E_i). The expected frequencies are calculated on the basis of our theoretical assumptions about the population distribution. Let us explain the procedure of testing by going through some illustrations.

Illustration 3

A sales man has 3 products to sell and there is a 40% chance of selling each product when he meets a customer. The following is the frequency distribution of sales.

No. of products sold per sale:	0	1	2	3
Frequency of the number of sales:	10	40	60	20

At the 0.05 level of significance, do these sales of products follow a binomial distribution?

Solution: In this illustration, the sales process is approximated by a binomial distribution with P=0.40 (with a 40% chance of selling each product).

Ho: The sales of three products has a binomial distribution with P=0.40.

H_1: The sales of three products do not have a binomial distribution with P=0.40.

Before we proceed further we must calculate the expected frequencies in order to determine whether the discrepancies between the observed frequencies and the expected frequencies (based on binomial distribution) should be ascribed to chance. We began determining the binomial probability in each situation of sales (0, 1, 2, 3 products sold per sale). For three products, we would find the probabilities of success by consulting the binomial probabilities Appendix Table-1. By looking at the column labelled as n = 3 and p = 0.40 we obtained the following figures of binomial probabilities of the sales.

No. of products sold per sale (r)	Binomial probabilities of the sales
0	0.216
1	0.432
2	0.288
3	0.064
	1.000

We now calculate the expected frequency of sales for each situation. There are 130 customers visited by the salesman. We multiply each probability by 130 (no. of customers visited) to arrive at the respective expected frequency. For example, 0.216 × 130 = 28.08.

The following table shows the observed frequencies and the expected frequencies.

No. of products sold per sale	Observed frequency	Binomial probability	Number of customers visited	Expected frequency
(1) (4)	(2)	(3)	(4)	(5) = (3) ×
0	10	0.216	130	28.08
1	40	0.432	130	56.16
2	60	0.288	130	37.44
3	20	0.064	130	8.32
Total	130			

Now we use the chi-square test to examine the significance of differences between observed frequencies and expected frequencies. The formula for calculating chi- square is

$$\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i}$$

The following table gives the calculation of chi-square.

Observed frequencies (O_i)	Expected frequencies (E_i)	$(O_i–E_i)$	$(O_i–E_i)^2$	$(O_i–E_i)^2/E_i$
10	28.08	–18.08	326.89	11.64
40	56.16	–16.16	261.15	4.65
60	37.44	22.56	508.95	13.59
20	8.32	11.68	136.42	16.40
130	130			χ^2 = 46.28

In order to draw inferences about this calculated value of χ^2 we are required to compare this with table value of χ^2. For this we need: (i) degrees of freedom (n-1), and (ii) level of significance. In the problem we are given that the level of significance is 0.05. The number of expected situations is 4. That is (0,1,2,3 products sold per sale) n = 4. Therefore, the degrees of freedom will be 3 (i.e., n-1 = 4–1 = 3). The table value from Appendix Table-4 is 7.815 at 3 degrees of freedom and 0.05 level of significance. Since the calculated value (χ^2 = 46.28) is greater than the table value (7.815), we reject the null hypothesis and accept the alternative hypothesis. We conclude that the observed frequencies do not follow the binomial distribution.

Let us take another illustration which relates to the normal distribution.

Illustration 4

In order to plan how much cash to keep on hand, a bank manager is interested in seeing whether the average deposit of a customer is normally distributed with mean Rs. 15000 and standard deviation Rs. 6000. The following information is available with the bank.

Deposit (Rs)	Less than 10000	10000-20000	More than 20000
Number of depositors	30	80	40

Calculate the χ^2 statistic and test whether the data follows a normal distribution with mean Rs.15000 and standard deviation Rs.6000 (take the level of significance
(a) as 0.10).

Solution: In this illustration, the assumption made by the bank manager is that the pattern of deposits follows a normal distribution with mean Rs.15000 and standard deviation Rs.6000. Therefore, in testing the goodness-of-fit you may like to state the following hypothesis.

H_0: The sample data of deposits is from a population having normal distribution with mean Rs.15000 and standard deviation Rs.6000.

H_1: The sample data of deposits is not from a population having normal distribution with mean Rs.15000 and standard deviation Rs.6000.

In order to calculate the χ^2 value we must have expected frequencies. The expected frequencies are determined by multiplying the proportion of population values within each class interval by the total sample size of observed frequencies. Since we have assumed a normal distribution for our population, the expected frequencies are calculated by multiplying the area under the respective normal curve and the total sample size (n=150).

For example, to obtain the area for deposits less than Rs.10000, we calculate the normal deviate as follows:

$$z = \frac{x-\mu}{\sigma} = \frac{10000-15000}{6000} = \frac{-5000}{6000} = -0.83$$

From Appendix Table-3 (given at the end of this unit), this value (–0.83) corresponds to a lower tail area of 0.5000–0.2967 = 0.2033. Multiplying 0.2033 by the sample size (150), we obtain the expected frequency 0.2033 × 150 = 30.50 depositors.
The calculations of the remaining expected frequencies are shown in the following table.

Upper limit of the deposit range (x)	Normal deviate $z = \frac{x-15000}{6000}$	Area left to x	Area of deposit range	Expected frequency (Depositors)
(1)	(2)	(3)	(4)	(5)=(4)×150
10000	–0.83	0.2033	0.2033	30.50
20000	0.83	0.7967	0.5934	89.01
>20000	∞	1.0000	0.2033	30.50
			1.0000	150

We should note that from Appendix Table-3 for 0.83 the area left to x is 0.5000 + 0.2967 = 0.7967 and for ∞ the area left to x is 0.5000 + 0.5000 = 1.0000. Similarly, the area of deposit range for normal deviate 0.83 = 0.7967–0.2033 = 0.5934 and for ∞ = 1.0000–0.7967 = 0.2033.

Once the expected frequencies are calculated, the procedure for calculating χ^2 statistic will be the same as we have seen in illustration 3.

$$\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i}$$

The following table gives the calculation of chi-square.

Observed frequencies(O_i)	Expected frequencies(E_i)	(O_i–E_i)	(O_i–E_i)^2	(O_i–E_i)^2/E_i
30	30.50	–0.50	0.2450	0.0080
80	89.01	–9.01	81.1801	0.9120
40	30.50	9.51	90.3450	2.9626
150	150			χ^2 = 3.8827

Since n = 3, the number of degrees of freedom will be n–1 = 3–1 = 2 and we are given 0.10 as the level of significance. From Appendix Table-4 the table value of χ^2 for df = 2 and α = 0.10 is 4.605. Since the calculated value of χ^2 (3.8827) is less than the table value we accept the null hypothesis and conclude that the data are well described by the normal distribution with mean = Rs.15000 and standard deviation = Rs. 6000.

Let us consider an illustration which relates to Poisson Distribution.

Illustration 5

A small car company wishes to determine the frequency distribution of warranty financed repairs per car for its new model car. On the basis of past experience the company believes that the pattern of repairs follows a Poisson distribution with mean number of repairs (l) as 3. A sample data of 400 observations is provided below:

No. of repairs more per car	0	1	2	3	4	5 or
No. of cars	20	57	98	85	78	62

i) Construct a table of expected frequencies using Poisson probabilities with l =3.

ii) Calculate the χ^2 statistic and give your conclusions about the null hypothesis (take the level of significance as 0.05).

Solution: For the above problem we formulate the following hypothesis.

H_0: The number of repairs per car during warranty period follows a Poisson probability distribution.

H_1: The number of repairs per car during warranty period does not follow a Poisson probability distribution.

As usual the expected frequencies are determined by multiplying the probability values (in this case Poisson probability) by the total sample size of observed frequencies. Appendix Table-2 provides the Poisson probability values. For λ = 3.0 and for different x values we can directly read the probability values. For example for λ = 3.0 and x = 0 the Poisson probability value is 0.0498, for λ = 3.0 and x = 1 the Poisson probability value is 0.1494 and so on … .

The following table gives the calculated expected frequencies.

No. of repairs per car (x)	Poisson probability	Expected frequency E_i = (2) × 400
(1)	(2)	(3)
0	0.0498	19.92
1	0.1494	59.76
2	0.2240	89.60
3	0.2240	89.60
4	0.1680	67.20
5 or more	0.1848	73.92
Total	1.0000	400

It is to be noted that from Appendix Table-2 for λ = 3.0 we have taken the Poisson probability values directly for x = 0,1,2,3 and 4. For x = 5 or more we added the rest of the probability values (for x = 5 to x = 12) so that the sum of all the probability for x = 0 to x = 5 or more will be 1.000.

As usual we use the following formula for calculating the chi-square (χ^2) value.

$$\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i}$$

The following table gives the calculated χ^2 value

Observed frequencies(O_i)	Expected frequencies(E_i)	$(O_i–E_i)$	$(O_i–E_i)^2$	$(O_i–E_i)^2/E_i$
20	19.92	0.08	0.0064	0.0003
57	59.76	– 2.76	7.6176	0.1275
98	89.60	8.40	70.5600	0.7875
85	89.60	– 4.60	21.1600	0.2362
78	67.20	10.80	116.6400	1.7357
62	73.92	– 11.92	142.0864	1.9222
400	400			χ^2 = 4.8094

Since n = 6, the number of degrees of freedom will be n–1 = 6–1 = 5 and we are given a = 0.05 as the level of significance. From table 4, the table value of χ^2 for 5 degrees of freedom and a = 0.05 is 11.071. Since the calculated value of χ^2 = 4.8094 which is less than the table value of χ^2 =11.071, we accept the null hypothesis (H_0) and conclude that the data follows a Poisson probability distribution with l = 3.0

Illustration 6

In order to know the brand preference of two washing detergents, a sample of 1000 consumers were surveyed. 56% of the consumers preferred Brand X and 44% of the consumers preferred Brand Y. Do these data conform to the idea that consumers have no special preference for either brand? Take significance level as 0.05.

Solution: In this illustration, we assume that brand preference follows a uniform distribution. That is, ½ of the consumers prefer Brand A and other ½ of the consumers prefer Brand B.

Therefore, we have the following hypothesis.

H_0: Brand name has no special significance for consumer preference.

H_1: Brand name has special significance for consumer preference.

Since the consumer preference data is given in proportion we will convert it into frequencies. The number of consumers who preferred Brand X are 0.56 × 1000 = 560 and Brand Y are 0.44 × 1000 = 440. The corresponding expected frequencies are ½ × 1000 = 500 each brand.

The following table gives calculated χ^2 value.

Observed frequencies(O_i)	Expected frequencies(E_i)	$(O_i–E_i)$	$(O_i–E_i)^2$	$(O_i–E_i)^2/E_i$
20	19.92	0.08	0.0064	0.0003
560	500	60	3600	7.2
440	500	– 60	3600	7.2
1000	1000			χ^2 = 14.4

The table value (by consulting the Appendix Table-4) at 5% significance level and n–1 = 2–1 = 1 degree of freedom is 3.841. Since the value of calculated χ^2 is 14.4 which is greater than table value, we reject the null hypothesis and conclude that the brand names have special significance for consumer preference.

17.5 CONDITIONS FOR APPLYING CHI-SQUARE TEST

To validate the chi-square test, the data set available, needs to fulfill certain conditions. Sometimes these conditions are also called precautions about using the chi-square test. Therefore, when ever you use the chi-square test the following conditions must be satisfied:

a) **Random Sample**: In chi-square test the data set used is assumed to be a random sample that represents the population. As with all significance tests, if you have a random sample data that represents population data, then any differences in the table values and the calculated values are real and therefore significant. On the other hand, if you have a non-random sample data, significance cannot be established, though the tests are nonetheless sometimes utilised as crude “rules of thumb” any way. For example, we reject the null hypothesis, if the difference between observed and expected frequencies is too large. But if the chi-square value is zero, we should be careful in interpreting that absolutely no difference exists between observed and expected frequencies. Then we should verify the quality of data collected whether the sample data represents the population or not.

b) **Large Sample Size**: To use the chi-square test you must have a large sample size that is enough to guarantee the test, to test the similarity between the theoretical distribution and the chi-square statistic. Applying chi-square test to small samples exposes the researcher to an unacceptable rate of type-II errors. However, there is no accepted cutoff sample size. Many researchers set the minimum sample size at 50. Remember that chi-square test statistic must be calculated on actual count data (nominal, ordinal or interval data) and not substituting percentages which would have the effect of projecting the sample size as 100.

c) **Adequate Cell Sizes**: You have seen above that small sample size leads to type-II error. That is, when the expected cell frequencies are too small, the value of chi-square will be overestimated. This in turn will result in too many rejections of the null hypothesis. To avoid making incorrect inferences from chi- square tests we follow a general rule that the expected frequency in any cell should be a minimum of 5.

d) **Independence**: The sample observations must be independent.

e) **Final values**: Observations must be grouped in categories.

17.6 CELLS POOLING

In the previous section we have seen that the cell size should be large enough of at least 5 or more. When a contingency table contains one or more cells with expected frequency of less than 5, this requirement may be met by combining two rows or columns before calculating χ^2. We must combine these cells in order to get an expected frequency of 5 or more in each cell. This practice is also known as grouping the frequencies together. But in doing this, we reduce the number of categories of data and will gain less information from contingency table. In addition, we also lose 1 or more degrees of freedom due to pooling. With this practice, it should be noted that the number of freedom is determined with the number of classes after the regrouping. In a special case 2 × 2 contingency table, the degree of freedom is 1. Suppose in any cell the frequency is less than 5, we may be tempted to apply the pooling method which results in 0 degrees of freedom (due to loss of 1 df) which is meaningless. When the assumption of cell frequency of minimum 5 is not maintained in case of a 2 × 2 contingency table we apply Yates correction. You will learn about Yates correction in section 17.7. Let us take an illustration to understand the cell pooling method.

Illustration 7

A company marketing manager wishes to determine whether there are any significant differences between regions in terms of a new product acceptance. The following is the data obtained from interviewing a sample of 190 consumers.

Degree of acceptance	Region				
	South	North	East	West	Total
Strong	30	25	20	30	105
Moderate	15	15	20	20	70
Poor	5	10	0	0	15
Total	50	50	40	50	190

Calculate the chi-square statistic. Test the independence of the two attributes at 0.05 degrees of freedom.

Solution: In this illustration, the null and alternate hypotheses are:

H_0: The product acceptance is independent of the region of the consumer.

H_1: The product acceptance is not independent of the region of the consumer.

We are given the observed frequencies in the problem. The following table gives the calculated expected frequencies.

Degree of acceptance	Region				
	South	North	East	West	Total
Strong	27.63	27.63	22.11	27.63	105
Moderate	18.42	18.42	14.74	18.42	70
Poor	3.95	3.95	3.16	3.95	15
Total	50.00	50.00	40.00	50.00	190

Since the expected frequencies (cell values) in the third row are less than 5 we pool the third row with the second row of both observed frequencies and expected frequencies. The revised observed frequency and expected frequency tables are given below.

Degree of acceptance	Region				
	South	North	East	West	Total
Strong	30	25	20	30	105
Moderate and poor	20	25	20	20	85
Total	50	50	40	50	190

Degree of acceptance	Region				
	South	North	East	West	Total
Strong	27.63	27.63	22.11	27.63	105
Moderate and poor	22.37	22.37	17.89	22.37	85
Total	50	50	40	50	190

Now we rearrange the data on observed and expected frequencies and calculate the χ^2 value. The following table gives the calculated χ^2 value.

(Row, Column)	Observed frequencies(O_i)	Expected frequencies(E_i)	(O_i-E_i)	$(O_i–E_i)^2$	$(O_i–E_i)^2/E_i$
(1,1)	30	27.63	2.37	5.6169	0.2033
(2,1)	20	22.37	–2.37	5.6169	0.2511
(1,2)	25	27.63	–2.63	6.9169	0.2503
(2,2)	25	22.37	2.63	6.9169	0.3092
(1,3)	20	22.11	–2.11	4.4521	0.2014
(2,3)	20	17.89	2.11	4.4521	0.2489
(1,4)	30	27.63	2.37	5.6169	0.2033
(2,4)	20	22.37	-2.37	5.6169	0.2511
					χ^2 =1.9185

Since we have a 2 × 4 contingency table, the degrees of freedom will be (r–1) × (c–1) = (2–1) × (4–1) = 1× 3 = 3. At 3 degree of freedom and 0.05 significance level the table value (from Appendix Table-4) is 7.815. Since the calculated χ^2 value (1.9185) is less than table value of χ^2 (7.815) we accept the null hypothesis and conclude that the product acceptance is independent of the region of the consumer.

Illustration 8

The following table gives the number of typing errors per page in a 40 page report. Test whether the typing errors per page have a Poisson distribution with mean (λ) number of errors is 3.0.

No. of typing errors per page	0	1	2	3	4	5	6	7	8	9	10 or more
No. of pages	5	9	6	8	4	3	2	1	1	0	1

i) Construct a table of expected frequencies using Poisson probabilities with $\lambda = 3$.

ii) Calculate the χ^2 statistic and give your conclusions about the null hypothesis (take the level of significance as 0.01).

Solution: For the above problem we formulate the following hypothesis.

H_0: The number of typing errors per page follows a Poisson probability distribution.

H_1: The number of typing errors per page does not follow a Poisson probability distribution.

As usual the expected frequencies are determined by multiplying the probability values (in this case Poisson probability) by the total sample size of observed frequencies. Table 17.3 provides the Poisson probability values. For $\lambda = 3.0$ and for different x values we can directly read the probability values. For example for $\lambda = 3.0$ and $x = 0$ the Poisson probability value is 0.0498. The following table gives the calculated expected frequencies.

No. of typing errors per page(x)	Poisson probability	Expected frequency $E_i = (2) \times 40$	
(1)	(2)	(3)	
0	0.0498	1.99	
1	0.1494	5.98	7.97
2	0.2240	8.96	
3	0.2240	8.96	
4	0.1680	6.72	
5	0.1008	4.03	
6	0.0504	2.02	
7	0.0216	0.86	
8	0.0081	0.32	14.11
9	0.0027	0.11	
10 or more	0.0012	0.05	
Total	1.0000	40	

Since the expected frequencies of the first row are less than 5, we pool first and second rows of observed and expected frequencies. Similarly, the expected frequencies of the last 6 rows (with 5,6,7,8,9, and 10 or more errors) are less than 5. Therefore we pool these rows with the row having the expected typing errors as 4 or more.

As usual we use the following formula for calculating the chi-square (χ^2) value.

$$\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i}$$

The following table gives the calculated χ^2 value after pooling cells

No. of typing errors per page (x)	Observed frequencies (O_i)	Expected frequencies (E_i)	(O_i-E_i)	$(O_i-E_i)^2$	$(O_i-E_i)^2/E_i$
1 or less	14	7.97	6.032	36.39	4.5664
2	6	8.96	–2.960	8.76	0.9779
3	8	8.96	–0.960	0.92	0.1029
4 or more	12	14.11	–2.112	4.46	0.3161
					χ^2 = 5.9632

Since n = 4, the number of degrees of freedom will be n–1 = 4–1 = 3 and we are given a = 0.01 as the level of significance. From Table 4 the table value of χ^2 for 3 degrees of freedom and a = 0.01 is 11.345. Since the calculated value of χ^2 = 5.9632 which is less than the table value of χ^2 =11.345, we accept the null hypothesis (H_0) and conclude that the typing errors follow a Poisson probability distribution with l = 3.0.

17.7 YATES CORRECTION

Yates correction is also called Yates correction for continuity. In a 2 x 2 contingency table the degrees of freedom is 1. If any one of the expected cell frequency is less than 5, then use of pooling method (explained in section 17.6) may result in 0 degree of freedom due to loss of 1 degree of freedom in pooling which is meaning less. More over, it is not valid to perform the chi square test if any one or more of the expected frequencies is less than 5 (as explained in section 17.5). Therefore, if any one or more of the expected frequencies in a 2 × 2 contingency table is less than 5, the Yates correction is applied. This was proposed by F. Yates, who was an English mathematician.

Suppose for a 2 × 2 contingency table, the four cell values a, b, c and d are arranged in the following order.

a	b
c	d

The Yates formula for corrected chi square is given by

$$\chi^2 = \frac{n\left[|ad - bc| - \frac{n}{2}\right]^2}{(a+b)(c+d)(a+c)(b+d)}$$

Illustration 9

Suppose we have the following data on the consumer preference of a new product collected from the people living in north and south India.

	South India	North India	Row total
Number of consumers who prefer present product	4	51	55
Number of consumers who prefer new product	14	38	52
Column total:	18	89	107

Do the data suggest that the new product is preferred by the people independent of their region? Use a = 0.05.

Solution: Suppose we symbolise the true proportions of people who prefer the new product as :

Ps = proportion of south Indians who prefer the new product

P_N = Proportion of north Indians who prefer the new product

We state the null hypothesis (H_0) and alternative hypothesis (H_1)as:

H_0: $P_S = P_N$ (the proportion of people who prefer new product among south and north India are the same).

H_1: $P_S \neq P_N$ (the proportion of people who prefer new product among south and north India are not the same).

In this illustration, (i) the sample size (n) = 107 (ii) the cell values are: a = 4, b = 51, c = 14, d = 38, (iii) The corresponding row totals are: (a + b) = 55 and (c + d) = 52, and column totals are (a + c) = 18 and (b + d) = 89.

Since one of the cell frequency is less than 5 (a = 4) we apply Yates correction to the chi-square test.

$$\chi^2 = \frac{n\left[|ad - bc| - \frac{n}{2}\right]^2}{(a+b)(c+d)(a+c)(b+d)}$$

$$\chi^2 = \frac{107\left[|4 \times 38 - 51 \times 14| - \frac{107}{2}\right]^2}{55 \times 52 \times 18 \times 89} = \frac{107\left[|152 - 714| - 53.5\right]^2}{4581720}$$

$$\chi^2 = \frac{107[562 - 53.5]^2}{4581720} = \frac{107[508.5]^2}{4581720}$$

$$\chi^2 = \frac{107 \times 258572}{4581720} = \frac{27667204}{4581720}$$

$$\therefore \chi^2 = 6.0386$$

The table value for degrees of freedom (2–1) (2–1) = 1 and significance level ∝ = 0.05 is 3.841. Since calculated value of chi-square is 6.0386 which is greater than the table value we can reject H_0 and accept H_1 and conclude that the preference for the new product is not independent of the geographical region.

It may be observed that when N is large, Yates correction will not make much difference in the chi square value. However, if N is small, the implication of Yates correction may overstate the probability.

17.8 LIMITATIONS OF CHI-SQUARE TEST

In order to prevent the misapplication of the χ^2 test, one has to keep the following limitations of the test in mind:

a) As explained in section 17.5 (conditions for applying chi square test), the chi square test is highly sensitive to the sample size. As sample size increases, absolute differences become a smaller and smaller proportion of expected value. This means that a reasonably strong association may not come up as significant if the sample size is small. Conversely, in a large sample, we may find statistical significance when the findings are small and insignificant. That is, the findings are not substantially significant, although they are statistically significant.

b) Chi-square test is also sensitive to small frequencies in the cells of contingency table. Generally, when the expected frequency in a cell of a table is less than 5, chi-square can lead to erroneous conclusions as explained in section 17.5. The rule of thumb here is that if either (i) an expected value in a cell in a 2×2 contingency table is less than 5 or (ii) the expected values of more than 20% of the cells in a greater than 2×2 contingency table are less than 5, then chi square test should not be applied. If at all a chi-square test is applied then appropriately either Yates correction or cell pooling should also be applied.

c) No directional hypothesis is assumed in chi-square test. Chi-square tests the hypothesis that two attributes/variables are related only by chance. That is if a significant relationship is found, this is not equivalent to establishing the researchers' hypothesis that attribute A causes attribute B or attribute B causes attribute A.

Self Assessment Exercise B

1) While calculating the expected frequencies of a chi-square distribution it was found that some of the cells of expected frequencies have value below 5. Therefore, some of the cells are pooled. The following statements tell you the size of the contingency table before pooling and the rows/columns pooled. Determine the number of degrees of freedom.

   a) 5×4 contingency table. First two and last two rows are pooled.

   b) 4×6 contingency table. First two and last two columns are pooled.

   c) 6×3 contingency table. First two rows are pooled. 4^{th}, 5^{th}, and 6^{th} rows are pooled.

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2) What is the table value of chi-square for goodness-of-fit if there are:

   a) 8 degrees of freedom and the significance level is 1%.

   b) 13 degrees of freedom and the significance level is 5%.

   c) 16 degrees of freedom and the significance level is 0.10%.

   d) 6 degrees of freedom and the significance level is 0.20%.

   ....

3) a) The following data is an observed frequency distribution. Assuming that the data follows a Poisson distribution with l=2.5.

   i) calculate Poisson probabilities and expected values, ii) calculate chi square value, and iii) at 0.05 level of significance can we conclude that the data follow a poisson distribution with l = 2.5.

No. of Telephone calls per minute	0	1	2	3	4	5 or more
Frequency of occurrences	6	30	41	52	12	9

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17.9 LET US SUM UP

There are several applications of chi-square distribution, some of which we have studied in this Unit. These are (i) to test the goodness-of-fit, and (ii) to test the independence of attributes. The chi-square distribution is known by its only parameter – number of degrees of freedom. Like student t distribution there is a separate chi-square distribution for each number of degrees of freedom.

The chi-square test for testing the goodness-of-fit establishes whether the sample data supports the assumption that a particular distribution applies to the parent population. It should be noted that the statistical procedures are based on some assumptions such as normal distribution of population. A chi-square procedure allows for testing the null hypothesis that a particular distribution applies. We also use chi-square test whether to test whether the classification criteria are independent or not.

When performing chi-square test using contingency tables, it is assumed that all cell frequencies are a minimum of 5. If this assumption is not met we may use the pooling method but then there is a loss of information when we use this method. In a 2 × 2 contingency table if one or more cell frequencies are less than 5 we should apply Yates correction for computing the chi-square value.

In a chi-square test for goodness of-fit, the degrees of freedom are number of categories – 1 (n–1). In a chi-square test for independence of attributes, the degrees of freedom are (number of rows–1) × (number of columns–1). That is, (r–1) × (c–1).

17.10 KEY WORDS

Adequate Cell Sizes: To avoid making incorrect inferences from chi-square tests we follow a general rule that the expected frequency in any cell should be a minimum of 7.

Cells Pooling: When a contingency table contains one or more cells with expected frequency less than 5, we combine two rows or columns before calculating χ^2. We combine these cells in order to get an expected frequency of 5 or more in each cell.

Chi-Square Distribution: A kind of probability distribution, differentiated by their degree of freedom, used to test a number of different hypotheses about variances, proportions and distributional goodness of fit.

Expected Frequencies: The hypothetical data in the cells are called as expected frequencies.

Goodness of Fit: The chi-square test procedure used for the validation of our assumption about the probability distribution is called goodness of fit.

Observed Frequencies: The actual cell frequencies are called observed frequencies.

Yates Correction: If any one or more of the expected frequencies in a 2 × 2 contingencies table is less than 5, the Yates correction is applied.

17.11 ANSWERS TO SELF ASSESSMENT EXERCISES

A) 1. a
- i) H_0: The preference for the type of car among people is independent of their sex.
- ii) degrees of freedom: 6
- iii) χ^2 (table value): 12.592
- iv) Conclusion: Accept H_0.

1. b
- i) H_0: The income distribution and preference for type of house are independently distributed.
- ii) degrees of freedom: 9
- iii) χ^2 (table value): 21.666
- iv) Conclusion: Reject H_0.

1. c
- i) H_0: The attitude towards going to a movie or for shopping is independent of the sex.
- ii) degrees of freedom: 1
- iii) χ^2 (table value): 6.635
- iv) Conclusion: Reject H_0.

1. d
- i) H_0: The voters educational level and their political affiliation are independent of each other.
- ii) degrees of freedom: 9
- iii) c^2 (table value): 14.684
- iv) Conclusion: Accept H_0.

2. a) 25, b) 6, c) 8, d) 21.

3. a.

(Row, Column)	Observed frequency (O_i)	Expected frequency (E_i)	$(O_i - E_i)$	$(O_i - E_i)^2$	$(O_i - E_i)^2/E_i$
(1,1)	50	52.5	–2.5	6.25	0.1190
(1,2)	55	52.5	2.5	6.25	0.1190
(1,3)	45	52.5	–7.5	56.25	1.0714
(1,4)	60	52.5	7.5	56.25	1.0714
(2,1)	50	47.5	2.5	6.25	0.1316
(2,2)	45	47.5	7.5	56.25	1.1842
(2,3)	55	47.5	–2.5	6.25	0.1316
(2,4)	40	47.5	12.5	156.25	3.2895
Total	400	400		$\chi^2 = 7.1178$	

3. b. H_0: The preference for the brand is distributed independent of the consumers' education level.

3. c. Table value χ^2 at 3 d.f and $\alpha = 0.05$ is 7.815. Since calculated value (7.1178) is less than the table value of χ^2 (7.815), we accept the H_0.

B) 1. a) 6, b) 9, c) 4

2. a) 20.090, b)22.362, c) 23.542, d) 8.558

3. i) Poisson probabilities and expected values

No. of repairs per car (x)	Poisson probability	Expected frequency E_i =(2)x150
(1)	(2)	(3)
0	0.0498	7.47
1	0.1494	22.41
2	0.2240	33.60
3	0.2240	33.60
4	0.1680	25.20
5 or more	0.1848	27.72

3. ii) chi-square value

No. of Telephone calls per minute	Observed frequency (O_i)	Expected frequency (E_i)	(O_i-E_i)	(O_i-E_i)^2	(O_i-E_i)^2/E_i
0	6	7.47	–1.47	2.16	0.2893
1	30	22.41	7.59	57.61	2.5706
2	41	33.60	7.40	54.76	1.6298
3	52	33.60	18.40	338.56	10.0762
4	12	25.20	–13.20	174.24	6.9143
5 or more	9	27.72	–18.72	350.44	12.6421
Total150	150				χ^2=34.1222

3.iii) At 0.05 significance level and 4 degrees of freedom the table value is 9.488. Since the calculated chi-square value is greater than the table value we reject the null hypothesis that the frequency of telephone calls follows Poisson distribution.

17.12 TERMINAL QUESTIONS/EXERCISES

1) Why do we use chi-square test?
2) What do you mean by expected frequencies in (a) chi-square test for testing independence of attributes, and (b) chi-square test for testing goodness-of-fit? Briefly explain the procedure you follow in calculating the expected values in each of the above situations.
3) Explain the conditions for applying chi-square test.
4) What are the limitations for applying chi-square test?
5) When do you use Yates correction?
6) When do you pool rows or columns while applying chi-square test? What are its limitations?
7) The following data provides information for 30 days on fatal accidents in a metro city. Do the data suggest that the distribution of fatal accidents follow a Poisson distribution? Take the level of significance as 0.05.

Fatal accidents per day	0	1	2	3	4 or more
Frequency	4	8	10	6	2

8) Below is an observed frequency distribution.

Marks range	Under 40	40 and under 50	50 and under 60	60 and under 75	75 and under 90	90 and above
No. of students	9	20	65	34	14	8

At 0.01 significance level, the null hypothesis is that the data is from normal distribution with a mean of 10 and a standard deviation of 2. What are your conclusions?

9) The following table gives the number of telephone calls attended by a credit card information attendant.

Day	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
No. of calls attended	45	50	24	36	33	27	42

Test whether the telephone calls are uniformly distributed? Use 0.10 significance level.

10) The following data gives preference of car makes by type of customer.

Type of customer	Car make				
	Maruti 800	Maruti Zen	Honda	Tata Indica	Total
Single man	350	200	150	50	750
Single woman	100	150	100	80	430
Married man	300	150	120	120	690
Married woman	150	100	80	50	380
Total	900	600	450	300	2250

(a) Test the independence of the two attributes. Use 0.05 level of significance.

(b) Draw your conclusions.

11) A bath soap manufacturer introduced a new brand of soap in 4 colours. The following data gives information on the consumer preference of the brand.

Consumer rating	Bath soap colour				
	Red	Green	Brown	Yellow	Total
Excellent	30	20	20	30	100
Good	20	10	20	30	80
Fair	20	10	10	30	70
Poor	10	45	35	10	100
Total	80	85	85	100	350

From the above data:

a) Compute the χ^2 value,

b) State the null hypothesis, and

c) Draw your inferences.

Note: These questions/exercises will help you to understand the unit better. Try to write answers for them. But do not submit your answers to the university for assessment. These are for your practice only.

17.13 FURTHER READING

A number of good text books are available on the topics dealth within this unit. The following books may be used for more indepth study.

1) Kothari, C.R.(1985) *Research Methodology Methods and Techniques*, Wiley Eastern, New Delhi.

2) Levin, R.I. and D.S. Rubin. (1999) *Statistics for Management*, Prentice-Hall of India, New Delhi

3) Mustafi, C.K.(1981) *Statistical Methods in Managerial Decisions*, Macmillan, New Delhi.

4) Chandan, J.S., *Statistics for Business and Economics,* Vikas Publishing House Pvt Ltd New Delhi.

5) Zikmund, William G. (1988) *Business Research Methods*, The Dryden Press, New York.

Appendix Table-1 Binomial Probabilities

		p																			
n	*r*	.01	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95
2	0	.980	.902	.810	.723	.640	.563	.490	.423	.360	.303	.250	.203	.160	.123	.090	.063	.040	.023	.010	.002
	1	.020	.095	.180	.255	.320	.375	.420	.455	.480	.495	.500	.495	.480	.455	.420	.375	.320	.255	.180	.095
	2	.000	.002	.010	.023	.040	.063	.090	.123	.160	.203	.250	.303	.360	.423	.490	.563	.640	.723	.810	.902
3	0	.970	.857	.729	.614	.512	.422	.343	.275	.216	.166	.125	.091	.064	.043	.027	.016	.008	.003	.001	.00
	1	.029	.135	.243	.325	.384	.422	.441	.444	.432	.408	.375	.334	.288	.239	.189	.141	.096	.057	.027	.007
	2	.000	.007	.027	.057	.096	.141	.189	.239	.288	.334	.375	.408	.432	.444	.441	.422	.384	.325	.243	.135
	3	.000	.000	.001	.003	.008	.016	.027	.043	.064	.091	.125	.166	.216	.275	.343	.422	.512	.614	.729	.857
4	0	.961	.815	.656	.522	.410	.316	.240	.179	.130	.092	.062	.041	.026	.015	.008	.004	.002	.001	.000	.000
	1	.039	.171	.292	.368	.410	.422	.412	.384	.346	.300	.250	.200	.154	.112	.076	.047	.026	.011	.004	.000
	2	.001	.014	.049	.098	.154	.211	.265	.311	.346	.368	.375	.368	.346	.311	.265	.211	.154	.098	.049	.014
	3	.000	.000	.004	.011	.026	.047	.076	.112	.154	.200	.250	.300	.346	.384	.412	.422	.410	.368	.292	.171
	4	.000	.000	.000	.001	.002	.004	.008	.015	.026	.041	.062	.092	.130	.179	.240	.316	.410	.522	.656	.815
5	0	.951	.774	.590	.444	.328	.237	.168	.116	.078	.050	.031	.019	.010	.005	.002	.001	.000	.000	.000	.000
	1	.048	.204	.328	.392	.410	.396	.360	.312	.259	.206	.156	.113	.077	.049	.028	.015	.006	.002	.000	.000
	2	.001	.021	.073	.138	.205	.264	.309	.336	.346	.337	.312	.276	.230	.181	.132	.088	.051	.024	.008	.001
	3	.000	.001	.008	.024	.051	.088	.132	.181	.230	.276	.312	.337	.346	.336	.309	.264	.205	.138	.073	.021
	4	.000	.000	.000	.002	.006	.015	.028	.049	.077	.113	.156	.206	.259	.312	.360	.396	.410	.392	.328	.204
	5	.000	.000	.000	.000	.000	.001	.002	.005	.010	.019	.031	.050	.078	.116	.168	.237	.328	.444	.590	.774
6	0	.941	.735	.531	.377	.262	.178	.118	.075	.047	.028	.016	.008	.004	.002	.001	.000	.000	.000	.000	.000
	1	.057	.232	.354	.399	.393	.356	.303	.244	.187	.136	.094	.061	.037	.020	.010	.004	.002	.000	.000	.000
	2	.001	.031	.098	.176	.246	.297	.324	.328	.311	.278	.234	.186	.138	.095	.060	.033	.015	.006	.001	.000
	3	.000	.002	.015	.042	.082	.132	.185	.236	.276	.303	.312	.303	.276	.236	.185	.132	.082	.042	.015	.002
	4	.000	.000	.001	.006	.015	.033	.060	.095	.138	.186	.234	.278	.311	.328	.324	.297	.246	.176	.098	.031
	5	.000	.000	.000	.000	.002	.004	.010	.020	.037	.061	.094	.136	.187	.244	.303	.356	.393	.399	.354	.232
	6	.000	.000	.000	.000	.000	.000	.001	.002	.004	.008	.016	.028	.047	.075	.118	.178	.262	.377	.531	.735
7	0	.932	.698	.478	.321	.210	.133	.082	.049	.028	.015	.008	.004	.002	.001	.000	.000	.000	.000	.000	.000
	1	.066	.257	.372	.396	.367	.311	.247	.185	.131	.087	.055	.032	.017	.008	.004	.001	.000	.000	.000	.000
	2	.002	.041	.124	.210	.275	.311	.318	.299	.261	.214	.164	.117	.077	.047	.025	.012	.004	.001	.000	.000
	3	.000	.004	.023	.062	.115	.173	.227	.268	.290	.292	.273	.239	.194	.144	.097	.058	.029	.011	.003	.000
	4	.000	.000	.003	.011	.029	.058	.097	.144	.194	.239	.273	.292	.290	;268	.227	.173	.115	.062	.023	.004
	5	.000	.000	.000	.001	.004	.012	.025	.047	.077	.117	.164	.214	.261	.299	.318	.311	.275	.210	.124	.041
	6	.000	.000	.000	.000	.000	.001	.004	.008	.017	.032	.055	.087	.131	.185	.247	.311	.367	.396	.372	.257
	7	.000	.000	.000	.000	.000	.000	.000	.001	.002	.004	.008	.015	.028	.049	.082	.133	.210	.321	.478	.698
8	0	.923	.663	.430	.272	.168	.100	.058	.032	.017	.008	.004	.002	.001	.000	.000	.000	.000	.000	.000	.000
	1	.075	.279	.383	.385	.336	.267	.198	.137	.090	.055	.031	.016	.008	.003	.001	.000	.000	.000	.000	.000
	2	.003	.051	.149	.238	.294	.311	.296	.259	.209	.157	.109	.070	.041	.022	.010	.004	.001	.000	.000	.000
	3	.000	.005	.033	.084	.147	.208	.254	.279	.279	.257	.219	.172	.124	.081	.047	.023	.009	.003	.000	.000
	4	.000	.000	.005	:018	.046	.087	.136	.188	.232	.263	.273	.263	.232	.188	.136	.087	.046	.018	.005	.000
	5	.000	.000	.000	.003	.009	.023	.047	.081	.124	.172	.219	.257	.279	.279	.254	.208	.147	.084	.033	.005
	6	.000	.000	.000	.000	.001	.004	.010	.022	.041	.070	.109	.157	.209	.259	.296	.311	.294	.238	.149	.051
	7	.000	.000	.000	.000	.000	.000	.001	.003	.008	.016	.031	.055	.090	.137	.198	.267	.336	.385	.383	.279
	8	.000	.000	.000	.000	.000	000	.000	.000	.001	.002	.004	.008	.017	.032	.058	.100	.168	.272	.430	.663

Appendix Table-1 Binomial Probabilities (continued)

n	r	p .01	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95
9	0	.914	.630	.387	.232	.134	.075	.040	.021	.010	.005	.002	.001	.000	.000	.000	.000	.000	.000	.000	.000
	1	.083	.299	.387	.368	.302	.225	.156	.100	.060	.034	.018	.008	.004	.001	.000	.000	.000	.000	.000	.000
	2	.003	.063	.172	.260	.302	.300	.267	.216	.161	.111	.070	.041	.021	.010	.004	.001	.000	.000	.000	.000
	3	.000	.008	.045	.107	.176	.234	.267	.272	.251	.212	.164	.116	.074	.042	.021	.009	.003	.001	.000	.000
	4	.000	.001	.007	.028	.066	.117	.172	.219	.251	.260	.246	.213	.167	.118	.074	.039	.017	.005	.001	.000
	5	.000	.000	.001	.005	.017	.039	.074	.118	.167	.213	.246	.260	.251	.219	.172	.117	.066	.028	.007	.001
	6	.000	.000	.000	.001	.003	.009	.021	.042	.074	.116	.164	.212	.251	.272	.267	.234	.176	.107	.045	.008
	7	.000	.000	.000	.000	.000	.001	.004	.010	.021	.041	.070	.111	.161	.216	.267	.300	.302	.260	.172	.063
	8	.000	.000	.000	.000	.000	.000	.000	.001	.004	.008	.018	.034	.060	.100	.156	.225	.302	.368	.387	.299
	9	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.002	.005	.010	.021	.040	.075	.134	.232	.387	.630
10	0	.904	.599	.349	.197	.107	.056	.028	.014	.006	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.091	.315	.387	.347	.268	.188	.121	.072	.040	.021	.010	.004	.002	.000	.000	.000	.000	.000	.000	.000
	2	.004	.075	.194	.276	.302	.282	.233	.176	.121	.076	.044	.023	.011	.004	.001	.000	.000	.000	.000	.000
	3	.000	.010	.057	.130	.201	.250	.267	.252	.215	.166	.117	.075	.042	.021	.009	.003	.001	.000	.000	.000
	4	.000	.001	.011	.040	.088	.146	.200	.238	.251	.238	.205	.160	.111	.069	.037	.016	.006	.001	.000	.000
	5	.000	.000	.001	.008	.026	.058	.103	.154	.201	.234	.246	.234	.201	.154	.103	.058	.026	.008	.001	.000
	6	.000	.000	.000	.001	.006	.016	.037	.069	.111	.160	.205	.238	.251	.238	.200	.146	.088	.040	.011	.001
	7	.000	.000	.000	.000	.001	.003	.009	.021	.042	.075	.117	.166	.215	.252	.267	.250	.201	.130	.057	.010
	8	.000	.000	.000	.000	.000	.000	.001	.004	.011	.023	.044	.076	.121	.176	.233	.282	.302	.276	.194	.07.
	9	.000	.000	.000	.000	.000	.000	.000	.000	.002	.004	.010	.021	.040	.072	.121	.188	.268	.347	.387	.315
	10	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.003	.006	.014	.028	.056	.107	.197	.349	.599
11	0	.895	.569	.314	.167	.086	.042	.020	.009	.004	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.099	.329	.384	.325	.236	.155	.093	.052	.027	.013	.005	.002	.001	.000	.000	.000	.000	.000	.000	.000
	2	.005	.087	.213	.287	.295	.258	.200	.140	.089	.051	.027	.013	.005	.002	.001	.000	.000	.000	.000	.000
	3	.000	.014	.071	.152	.221	.258	.257	.225	.177	.126	.081	.046	.023	.010	.004	.001	.000	.000	.000	.000
	4	.000	.001	.016	.054	.111	.172	.220	.243	.236	.206	.161	.113	.070	.038	.017	.006	.002	.000	.000	.000
	5	.000	.000	.002	.013	.039	.080	.132	.183	.221	.236	.226	.193	.147	.099	.057	.027	.010	.002	.000	.000
	6	.000	.000	.000	.002	.010	.027	.057	.099	.147	.193	.226	.236	.221	.183	.132	.080	.039	.013	.002	.000
	7	.000	.000	.000	.000	.002	.006	.017	.038	.070	.113	.161	.206	.236	.243	.220	.172	.111	.054	.016	.001
	8	.000	.000	.000	.000	.000	.001	.004	.010	.023	.046	.081	.126	.177	.225	.257	.258	.221	.152	.071	.014
	9	.000	.000	.000	.000	.000	.000	.001	.002	.005	.013	.027	.051	.089	.140	.200	.258	.295	.287	.213	.087
	10	.000	.000	.000	.000	.000	.000	.000	.000	.001	.002	.005	.013	.027	.052	.093	.155	.236	.325	.384	.329
	11	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.004	.009	.020	.042	.086	.167	.314	.569
12	0	.886	.540	.282	.142	.069	.032	.014	.006	.002	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.107	.341	.377	.301	.206	.127	.071	.037	.017	.008	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000
	2	.006	.099	.230	.292	.283	.232	.168	.109	.064	.034	.016	.007	.002	.001	.000	.000	.000	.000	.000	.000
	3	.000	.017	.085	.172	.236	.258	.240	.195	.142	.092	.054	.028	.012	.005	.001	.000	.000	.000	.000	.000
	4	.000	.002	.021	.068	.133	.194	.231	.237	.213	.170	.121	.076	.042	.020	.008	.002	.001	.000	.000	.000
	5	.000	.000	.004	.019	.053	.103	.158	.204	.227	.223	.193	.149	.101	.059	.029	.011	.003	.001	.000	.000
	6	.000	.000	.000	.004	.016	.040	.079	.128	.177	.212	.226	.212	.177	.128	.079	.040	.016	.004	.000	.000

Appendix Table-1 Binomial Probabilities (continued)

		p																			
n	*r*	.01	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95
	7	.000	.000	.000	.001	.003	.011	.029	.059	.101	.149	.193	.223	.227	.204	.158	.103	.053	.019	.004	.000
	8	.000	.000	.000	.000	.001	.002	.008	.020	.042	.076	.121	.170	.213	.237	.231	.194	.133	.068	.021	.002
	9	.000	.000	.000	.000	.000	.000	.001	.005	.012	.028	.054	.092	.142	.195	.240	.258	.236	.172	.085	.017
	10	.000	.000	.000	.000	.000	.000	.000	.001	.002	.007	.016	.034	.064	.109	.168	.232	.283	.292	.230	.099
	11	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.003	.008	.017	.037	.071	.127	.206	.301	.377	.341
	12	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.002	.006	.014	.032	.069	.142	.282	.540
15	0	.860	.463	.206	.087	.035	.013	.005	.002	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.130	.366	.343	.231	.132	.067	.031	.013	.005	.002	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	2	.009	.135	.267	.286	.231	.156	.092	.048	.022	.009	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000
	3	.000	.031	.129	.218	.250	.225	.170	.111	.063	.032	.014	.005	.002	.000	.000	.000	.000	.000	.000	.000
	4	.000	.005	.043	.116	.188	.225	.219	.179	.127	.078	.042	.019	.007	.002	.001	.000	.000	.000	.000	.000
	5	.000	.001	.010	.045	.103	.165	.206	.212	.186	.140	.092	.051	.024	.010	.003	.001	.000	.000	.000	.000
	6	.000	.000	.002	.013	.043	.092	.147	.191	.207	.191	.153	.105	.061	.030	.012	.003	.001	.000	.000	.000
	7	.000	.000	.000	.003	.014	.039	.081	.132	.177	.201	.196	.165	.118	.071	.035	.013	.003	.001	.000	.000
	8	.000	.000	.000	.001	.003	.013	.035	.071	.118	.165	.196	.201	.177	.132	.081	.039	.014	.003	.000	.000
	9	.000	.000	.000	.000	.001	.003	.012	.030	.061	.105	.153	.191	.207	.191	.147	.092	.043	.013	.002	.000
	10	.000	.000	.000	.000	.000	.001	.003	.010	.024	.051	.092	.140	.186	.212	.206	.165	.103	.045	.010	.001
	11	.000	.000	.000	.000	.000	.000	.001	.002	.007	.019	.042	.078	.127	.179	.219	.225	.188	.116	.043	.005
	12	.000	.000	.000	.000	.000	.000	.000	.000	.002	.005	.014	.032	.063	.111	.170	.225	.250	.218	.129	.031
	13	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.003	.009	.022	.048	.092	.156	.231	.286	.267	.135
	14	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.002	.005	.013	.031	.067	.132	.231	.343	.366
	15	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.002	.005	.013	.035	.087	.206	.463

Appendix Table-2 Direct Values for Determining Poisson Probabilities

For a given value of l, entry indicates the probability of obtaining a specified value of X.

μ

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679
1	0.0905	0.1637	0.2222	0.2681	0.3033	0.3293	0.3476	0.3595	0.3659	0.3679
2	0.0045	0.0164	0.0333	0.0536	0.0758	0.0688	0.1217	0.1438	0.1647	0.1839
3	0.0002	0.0011	0.0033	0.0072	0.0126	0.0198	0.0284	0.0383	0.0494	0.0613
4	0.0000	0.0001	0.0003	0.0007	0.0016	0.0030	0.0050	0.0077	0.0111	0.0153
5	0.0000	0.0000	0.0000	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020	0.0031
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0003	0.0005
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001

μ

x	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496	0.1353
1	0.3662	0.3614	0.3543	0.3452	0.3347	0.3230	0.3106	0.2975	0.2842	0.2707
2	0.2014	0.2169	0.2303	0.2417	0.2510	0.2584	0.2640	0.2678	0.2700	0.2707
3	0.0738	0.0867	0.0998	0.1128	0.1255	0.1378	0.1496	0.1607	0.1710	0.1804
4	0.0203	0.0260	0.0324	0.0395	0.0471	0.0551	0.0636	0.0723	0.0812	0.0902
5	0.0045	0.0062	0.0084	0.0111	0.0141	0.0176	0.0216	0.0260	0.0309	0.0361
6	0.0008	0.0012	0.0018	0.0026	0.0035	0.0047	0.0061	0.0078	0.0098	0.0120
7	0.0001	0.0002	0.0003	0.0005	0.0008	0.0011	0.0015	0.0020	0.0027	0.0034
8	0.0000	0.0000	0.0001	0.0001	0.0001	0.0002	0.0003	0.0005	0.0006	0.0009
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0002

μ

x	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
0	0.1225	0.1108	0.1003	0.0907	0.0821	0.0743	0.0672	0.0608	0.0550	0.0498
1	0.2572	0.2438	0.2306	0.2177	0.2052	0.1931	0.1815	0.1703	0.1596	0.1494
2	0.2700	0.2681	0.2652	0.5613	0.2565	0.2510	0.2450	0.2384	0.2314	0.2240
3	0.1890	0.1966	0.2033	0.2090	0.2138	0.2176	0.2205	0.2225	0.2237	0.2240
4	0.0992	0.1082	0.1169	0.1254	0.1336	0.1414	0.1488	0.1557	0.1622	0.1680
5	0.0417	0.0476	0.0538	0.0602	0.0668	0.0735	0.0804	0.0872	0.0940	0.1008
6	0.0146	0.0174	0.0206	0.0241	0.0278	0.0319	0.0362	0.0407	0.0455	0.0504
7	0.0044	0.0055	0.0068	0.0083	0.0099	0.0118	0.0139	0.0163	0.0188	0.0216
8	0.0011	0.0015	0.0019	0.0025	0.0031	0.0038	0.0047	0.0057	0.0068	0.0081
9	0.0003	0.0004	0.0005	0.0007	0.0009	0.0011	0.0014	0.0018	0.0022	0.0027
10	0.0001	0.0001	0.0001	0.0002	0.0002	0.0003	0.0004	0.0005	0.0006	0.0008
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0002	0.0002
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001

μ

x	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
0	0.0450	0.0408	0.0369	0.0334	0.0302	0.0273	0.0247	0.0224	0.0202	0.0183
1	0.1397	0.1304	0.1217	0.1135	0.1057	0.0984	0.0915	0.0850	0.0789	0.0733
2	0.2165	0.2087	0.2008	0.1929	0.1850	0.1771	0.1692	0.1615	0.1539	0.1465
3	0.2237	0.2226	0.2209	0.2186	0.2158	0.2125	0.2087	0.2046	0.2001	0.1954
4	0.1734	0.1781	0.1823	0.1858	0.1888	0.1912	0.1931	0.1944	0.1951	0.1954
5	0.1075	0.1140	0.1203	0.1264	0.1322	0.1377	0.1429	0.1477	0.1522	0.1563
6	0.0555	0.0608	0.0662	0.0716	0.0771	0.0826	0.0881	0.0936	0.0989	0.1042
7	0.0246	0.0278	0.0312	0.0348	0.0385	0.0425	0.0466	0.0508	0.0551	0.0595
8	0.0095	0.0111	0.0129	0.0148	0.0169	0.0191	0.0215	0.0241	0.0269	0.0298
9	0.0033	0.0040	0.0047	0.0056	0.0066	0.0076	0.0089	0.0102	0.0116	0.0132
10	0.0010	0.0013	0.0016	0.0019	0.0023	0.0028	0.0033	0.0039	0.0045	0.0053
11	0.0003	0.0004	0.0005	0.0006	0.0007	0.0009	0.0011	0.0013	0.0016	0.0019
12	0.0001	0.0001	0.0001	0.0002	0.0002	0.0003	0.0003	0.0004	0.0005	0.0006
13	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001

Appendix Table-2 Direct Values for Determining Poisson Probabilities (continued….)

	μ									
x	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
0	0.0166	0.0150	0.0136	0.0123	0.0111	0.0101	0.0091	0.0082	0.0074	0.0067
1	0.0679	0.0630	0.0583	0.0540	0.0500	0.0462	0.0427	0.0395	0.0365	0.0337
2	0.1393	0.1323	0.1254	0.1188	0.1125	0.1063	0.1005	0.0948	0.0894	0.0842
3	0.1904	0.1852	0.1798	0.1743	0.1687	0.1631	0.1574	0.1517	0.1460	0.1404
4	0.1951	0.1944	0.1933	0.1917	0.1898	0.1875	0.1849	0.1820	0.1789	0.1755
5	0.1600	0.1633	0.1662	0.1687	0.1708	0.1725	0.1738	0.1747	0.1753	0.1755
6	0.1093	0.1143	0.1191	0.1237	0.1281	0.1323	0.1362	0.1398	0.1432	0.1462
7	0.0640	0.0686	0.0732	0.0778	0.0824	0.0869	0.0914	0.0959	0.1022	0.1044
8	0.0328	0.0360	0.0393	0.0428	0.0463	0.0500	0.0537	0.0575	0.0614	0.0653
9	0.0150	0.0168	0.0188	0.0209	0.0232	0.0255	0.0280	0.0307	0.0334	0.0363
10	0.0061	0.0071	0.0081	0.0092	0.0104	0.0118	0.0132	0.0147	0.0164	0.0181
11	0.0023	0.0027	0.0032	0.0037	0.0043	0.0049	0.0056	0.0064	0.0073	0.0082
12	0.0008	0.0009	0.0011	0.0014	0.0016	0.0019	0.0022	0.0026	0.0030	0.0034
13	0.0002	0.0003	0.0004	0.0005	0.0006	0.0007	0.0008	0.0009	0.0011	0.0013
14	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0003	0.0003	0.0004	0.0005
15	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.000

	μ									
x	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0
0	0.0061	0.0055	0.0050	0.0045	0.0041	0.0037	0.0033	0.0030	0.0027	0.0025
1	0.0311	0.0287	0.0265	0.0244	0.0225	0.0207	0.0191	0.0176	0.0162	0.0149
2	0.0793	0.0746	0.0701	0.0659	0.0618	0.0580	0.0544	0.0509	0.0477	0.0446
3	0.1348	0.1293	0.1239	0.1185	0.1133	0.1082	0.1033	0.0985	0.0938	0.0892
4	0.1719	0.1681	0.1641	0.1600	0.1558	0.1515	0.1472	0.1428	0.1383	0.1339
5	0.1753	0.1748	0.1740	0.1728	0.1714	0.1697	0.1678	0.1656	0.1632	0.1606
6	0.1490	0.1515	0.1537	0.1555	0.1571	0.1584	0.1594	0.1601	0.1605	0.1606
7	0.1086	0.1125	0.1163	0.1200	0.1234	0.1267	0.1298	0.1326	0.1353	0.1377
8	0.0692	0.0731	0.0771	0.0810	0.0849	0.0887	0.0925	0.0962	0.0998	0.1033
9	0.0392	0.0423	0.0454	0.0486	0.0519	0.0552	0.0586	0.0620	0.0654	0.0688
10	0.0200	0.0220	0.0241	0.0262	0.0285	0.0309	0.0334	0.0359	0.0386	0.0413
11	0.0093	0.0104	0.0116	0.0129	0.0143	0.0157	0.0173	0.0190	0.0207	0.0225
12	0.0039	0.0045	0.0051	0.0058	0.0065	0.0073	0.0082	0.0092	0.0102	0.0113
13	0.0015	0.0018	0.0021	0.0024	0.0028	0.0032	0.0036	0.0041	0.0046	0.0052
14	0.0006	0.0007	0.0008	0.0009	0.0011	0.0013	0.0015	0.0017	0.0019	0.0022
15	0.0002	0.0002	0.0003	0.0003	0.0004	0.0005	0.0006	0.0007	0.0008	0.0009
16	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0003	0.0003
17	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001

	μ									
x	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0
0	0.0022	0.0020	0.0018	0.0017	0.0015	0.0014	0.0012	0.0011	0.0010	0.0009
1	0.0137	0.0126	0.0116	0.0106	0.0098	0.0090	0.0082	0.0076	0.0070	0.0064
2	0.0417	0.0390	0.0364	0.0340	0.0318	0.0296	0.0276	0.0258	0.0240	0.0223
3	0.0848	0.0806	0.0765	0.0726	0.0688	0.0652	0.0617	0.0584	0.0552	0.0521
4	0.1294	0.1249	0.1205	0.1162	0.1118	0.1076	0.1034	0.0992	0.0952	0.0912
5	0.1579	0.1549	0.1519	0.1487	0.1454	0.1420	0.1385	0.1349	0.1314	0.1277
6	0.1605	0.1601	0.1595	0.1586	0.1575	0.1562	0.1546	0.1529	0.1511	0.1490
7	0.1399	0.1418	0.1435	0.1450	0.1462	0.1472	0.1480	0.1486	0.1489	0.1490
8	0.1066	0.1099	0.1130	0.1160	0.1188	0.1215	0.1240	0.1263	0.1284	0.1304
9	0.0723	0.0757	0.0791	0.0825	0.0858	0.0891	0.0923	0.0954	0.0985	0.1014
10	0.0441	0.0469	0.0498	0.0528	0.0558	0.0588	0.0618	0.0649	0.0679	0.0710
11	0.0245	0.0265	0.0285	0.0307	0.0330	0.0353	0.0377	0.0401	0.0426	0.0452
12	0.0124	0.0137	0.0150	0.0164	0.0179	0.0194	0.0210	0.0227	0.0245	0.0264
13	0.0058	0.0065	0.0073	0.0081	0.0089	0.0098	0.0108	0.0119	0.0130	0.0142
14	0.0025	0.0029	0.0033	0.0037	0.0041	0.0046	0.0052	0.0058	0.0064	0.0071
15	0.0010	0.0012	0.0014	0.0016	0.0018	0.0020	0.0023	0.0026	0.0029	0.0033
16	0.0004	0.0005	0.0005	0.0006	0.0007	0.0008	0.0010	0.0011	0.0013	0.0014
17	0.0001	0.0002	0.0002	0.0002	0.0003	0.0003	0.0004	0.0004	0.0005	0.0006
18	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002
19	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001

	μ									
x	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0
0	0.0008	0.0007	0.0007	0.0006	0.0006	0.0005	0.0005	0.0004	0.0004	0.0003
1	0.0059	0.0054	0.0049	0.0045	0.0041	0.0038	0.0035	0.0032	0.0029	0.0027
2	0.0208	0.0194	0.0180	0.0167	0.0156	0.0145	0.0134	0.0125	0.0116	0.0107
3	0.0492	0.0464	0.0438	0.0413	0.0389	0.0366	0.0345	0.0324	0.0305	0.0286
4	0.0874	0.0836	0.0799	0.0764	0.0729	0.0696	0.0663	0.0632	0.0602	0.0573
5	0.1241	0.1204	0.1167	0.1130	0.1094	0.1057	0.1021	0.0986	0.0951	0.0916
6	0.1468	0.1445	0.1420	0.1394	0.1367	0.1339	0.1311	0.1282	0.1252	0.1221
7	0.1489	0.1486	0.1481	0.1474	0.1465	0.1454	0.1442	0.1428	0.1413	0.1396
8	0.1321	0.1337	0.1351	0.1363	0.1373	0.1382	0.1388	0.1392	0.1395	0.1396
9	0.1042	0.1070	0.1096	0.1121	0.1144	0.1167	0.1187	0.1207	0.1224	0.1241
10	0.0740	0.0770	0.0800	0.0829	0.0858	0.0887	0.0914	0.0941	0.0967	0.0993
11	0.0478	0.0504	0.0531	0.0558	0.0585	0.0613	0.0640	0.0667	0.0695	0.0722
12	0.0283	0.0303	0.0323	0.0344	0.0366	0.0388	0.0411	0.0434	0.0457	0.0481
13	0.0154	0.0168	0.0181	0.0196	0.0211	0.0227	0.0243	0.0260	0.0278	0.0296
14	0.0078	0.0086	0.0095	0.0104	0.0113	0.0123	0.0134	0.0145	0.0157	0.0169
15	0.0037	0.0041	0.0046	0.0051	0.0057	0.0062	0.0069	0.0075	0.0083	0.0090
16	0.0016	0.0019	0.0021	0.0024	0.0026	0.0030	0.0033	0.0037	0.0041	0.0045
17	0.0007	0.0008	0.0009	0.0010	0.0012	0.0013	0.0015	0.0017	0.0019	0.0021
18	0.0003	0.0003	0.0004	0.0004	0.0005	0.0006	0.0006	0.0007	0.0008	0.0009
19	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003	0.0004
20	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002
21	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001

	μ									
x	8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9.0
0	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001
1	0.0025	0.0023	0.0021	0.0019	0.0017	0.0016	0.0014	0.0013	0.0012	0.0011
2	0.0100	0.0092	0.0086	0.0079	0.0074	0.0068	0.0063	0.0058	0.0054	0.0050
3	0.0269	0.0252	0.0237	0.0222	0.0208	0.0195	0.0183	0.0171	0.0260	0.0150
4	0.0544	0.0517	0.0491	0.0466	0.0443	0.0420	0.0398	0.0377	0.0357	0.0337
5	0.0882	0.0849	0.0816	0.0784	0.0752	0.0722	0.0692	0.0663	0.0635	0.0607
6	0.1191	0.1160	0.1128	0.1097	0.1066	0.1034	0.1003	0.0972	0.0941	0.0911
7	0.1378	0.1358	0.1338	0.1317	0.1294	0.1271	0.1247	0.1222	0.1197	0.1171
8	0.1395	0.1392	0.1388	0.1382	0.1375	0.1366	0.1356	0.1344	0.1332	0.1318
9	0.1256	0.1269	0.1280	0.1290	0.1299	0.1306	0.1311	0.1315	0.1317	0.1318
10	0.1017	0.1040	0.1063	0.1084	0.1104	0.1123	0.1140	0.1157	0.1172	0.1186
11	0.0749	0.0776	0.0802	0.0828	0.0853	0.0878	0.0902	0.0925	0.0948	0.0970
12	0.0505	0.0530	0.0555	0.0579	0.0604	0.0629	0.0654	0.0679	0.0703	0.0728
13	0.0315	0.0334	0.0354	0.0374	0.0395	0.0416	0.0438	0.0459	0.0481	0.0504
14	0.0182	0.0196	0.0210	0.0225	0.0240	0.0256	0.0272	0.0289	0.0306	0.0324
15	0.0098	0.0107	0.0116	0.0126	0.0136	0.0147	0.0158	0.0169	0.0182	0.0194
16	0.0050	0.0055	0.0060	0.0066	0.0072	0.0079	0.0086	0.0093	0.0101	0.0109
17	0.0024	0.0026	0.0029	0.0033	0.0036	0.0040	0.0044	0.0048	0.0053	0.0058
18	0.0011	0.0012	0.0014	0.0015	0.0017	0.0019	0.0021	0.0024	0.0026	0.0029
19	0.0005	0.0005	0.0006	0.0007	0.0008	0.0009	0.0010	0.0011	0.0012	0.0014
20	0.0002	0.0002	0.0002	0.0003	0.0003	0.0004	0.0004	0.0005	0.0005	0.0006
21	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0003
22	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001

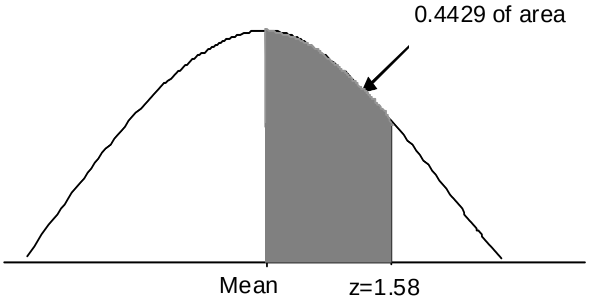
	μ									
x	9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10.0
0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000
1	0.0010	0.0009	0.0009	0.0008	0.0007	0.0007	0.0006	0.0005	0.0005	0.0005
2	0.0046	0.0043	0.0040	0.0037	0.0034	0.0031	0.0029	0.0027	0.0025	0.0023
3	0.1040	0.0131	0.0123	0.0115	0.0107	0.0100	0.0093	0.0087	0.0081	0.0076
4	0.0319	0.0302	0.0285	0.0269	0.0254	0.0240	0.0226	0.0213	0.0201	0.0189

Appendix Table-2 Direct Values for Determining Poisson Probabilities (continued….)

5	0.0581	0.0555	0.0530	0.0506	0.0483	0.0460	0.0439	0.0418	0.0398	0.0378
6	0.0881	0.0851	0.0822	0.0793	0.0764	0.0736	0.0709	0.0682	0.0656	0.0631
7	0.1145	0.1118	0.1091	0.1064	0.1037	0.1010	0.0982	0.0955	0.0928	0.0901
8	0.1302	0.1286	0.1269	0.1251	0.1232	0.1212	0.1191	0.1170	0.1148	0.1126
9	0.1317	0.1315	0.1311	0.1306	0.1300	0.1293	0.1284	0.1274	0.1263	0.1251
10	0.1198	0.1210	0.1219	0.1228	0.1235	0.1241	0.1245	0.1249	0.1250	0.1251
11	0.0991	0.1012	0.1031	0.1049	0.1067	0.1083	0.1098	0.1112	0.1125	0.1137
12	0.0752	0.0776	0.0799	0.0822	0.0844	0.0866	0.0888	0.0908	0.0928	0.0948
13	0.0526	0.0549	0.0572	0.0594	0.0617	0.0640	0.0662	0.0685	0.0707	0.0729
14	0.0342	0.0361	0.0380	0.0399	0.0419	0.0439	0.0459	0.0479	0.0500	0.0521
15	0.0208	0.0221	0.0235	0.0250	0.0265	0.0281	0.0297	0.0313	0.0330	0.0347
16	0.0118	0.0127	0.0137	0.0147	0.0157	0.0168	0.0180	0.0192	0.0204	0.0217
17	0.0063	0.0069	0.0075	0.0081	0.0088	0.0095	0.0103	0.0111	0.0119	0.0128
18	0.0032	0.0035	0.0039	0.0042	0.0046	0.0051	0.0055	0.0060	0.0065	0.0071
19	0.0015	0.0017	0.0019	0.0021	0.0023	0.0026	0.0028	0.0031	0.0034	0.0037
20	0.0007	0.0008	0.0009	0.0010	0.0011	0.0012	0.0014	0.0015	0.0017	0.0019
21	0.0003	0.0003	0.0004	0.0004	0.0005	0.0006	0.0006	0.0007	0.0008	0.0009
22	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	0.0004	0.0004
23	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
24	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001

	μ									
x	11	12	13	14	15	16	17	18	19	20
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0010	0.0004	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0037	0.0018	0.0008	0.0004	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
4	0.0102	0.0053	0.0027	0.0013	0.0006	0.0003	0.0001	0.0001	0.0000	0.0000
5	0.0224	0.0127	0.0070	0.0037	0.0019	0.0010	0.0005	0.0002	0.0001	0.0001
6	0.0411	0.0255	0.0152	0.0087	0.0048	0.0026	0.0014	0.0007	0.0004	0.0002
7	0.0646	0.0437	0.0281	0.0174	0.0104	0.0060	0.0034	0.0018	0.0010	0.0005
8	0.0888	0.0655	0.0457	0.0304	0.0194	0.0120	0.0072	0.0042	0.0024	0.0013
9	0.1085	0.0874	0.0661	0.0473	0.0324	0.0213	0.0135	0.0083	0.0050	0.0029
10	0.1194	0.1048	0.0859	0.0663	0.0486	0.0341	0.0230	0.0150	0.0095	0.0058
11	0.1194	0.1144	0.1015	0.0844	0.0663	0.0496	0.0355	0.0245	0.0164	0.0106
12	0.1094	0.1144	0.1099	0.0984	0.0829	0.0661	0.0504	0.0368	0.0259	0.0176
13	0.0926	0.1056	0.1099	0.1060	0.0956	0.0814	0.0658	0.0509	0.0378	0.0271
14	0.0728	0.0905	0.1021	0.1060	0.1024	0.0930	0.0800	0.0655	0.0514	0.0387
15	0.0534	0.0724	0.0885	0.0989	0.1024	0.0992	0.0906	0.0786	0.0650	0.0516
16	0.0367	0.0543	0.0719	0.0866	0.0960	0.0992	0.0963	0.0884	0.0772	0.0646
17	0.0237	0.0383	0.0550	0.0713	0.0847	0.0934	0.0963	0.0936	0.0863	0.0760
18	0.0145	0.0256	0.0397	0.0554	0.0706	0.0830	0.0909	0.0936	0.0911	0.0844
19	0.0084	0.0161	0.0272	0.0409	0.0557	0.0699	0.0814	0.0887	0.0911	0.0888
20	0.0046	0.0097	0.0177	0.0286	0.0418	0.0559	0.0692	0.0798	0.0866	0.0888
21	0.0024	0.0055	0.0109	0.0191	0.0299	0.0426	0.0560	0.0684	0.0783	0.0846
22	0.0012	0.0030	0.0065	0.0121	0.0204	0.0310	0.0433	0.0560	0.0676	0.0769
23	0.0006	0.0016	0.0037	0.0074	0.0133	0.0216	0.0320	0.0438	0.0559	0.0669
24	0.0003	0.0008	0.0020	0.0043	0.0083	0.0144	0.0226	0.0328	0.0442	0.0557
25	0.0001	0.0004	0.0010	0.0024	0.0050	0.0092	0.0154	0.0237	0.0336	0.0446
26	0.0000	0.0002	0.0005	0.0013	0.0029	0.0057	0.0101	0.0164	0.0246	0.0343
27	0.0000	0.0001	0.0002	0.0007	0.0016	0.0034	0.0063	0.0109	0.0173	0.0254
28	0.0000	0.0000	0.0001	0.0003	0.0009	0.0019	0.0038	0.0070	0.0117	0.0181
29	0.0000	0.0000	0.0001	0.0002	0.0004	0.0011	0.0023	0.0044	0.0077	0.0125
30	0.0000	0.0000	0.0000	0.0001	0.0002	0.0006	0.0013	0.0026	0.0049	0.0083
32	0.0000	0.0000	0.0000	0.0000	0.0001	0.0003	0.0007	0.0015	0.0030	0.0054
32	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0004	0.0009	0.0018	0.0034
33	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0005	0.0010	0.0020
34	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0006	0.0012
35	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0003	0.0007
36	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0004
37	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002
38	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
39	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001

Appendix Table-3 Areas of a Standard Normal Probability Distribution Between the Mean and Positive Values of z.



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	:4821	.4826	:4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999

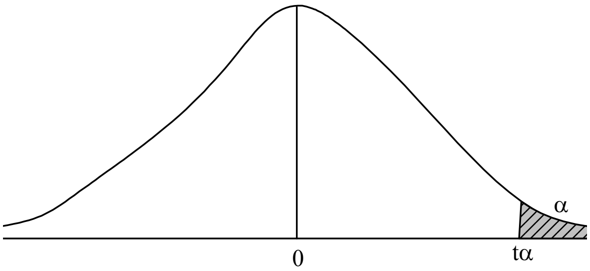
Appendix Table-4 Area in the Right Tail of a Chi-Square (χ^2) Distribution

Degrees of freedom	Area in right tail									
	.99	.975	.95	.90	.80	.20	.10	.05	.025	.01
1	0.00016	0.00098	0.00393	0.0158	0.0642	1.642	2.706	3.841	5.024	6.635
2	0.0201	0.0506	0.103	0.211	0.446	3.219	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	1.005	4.642	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	1.649	5.989	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.610	2.343	7.289	9.236	11.071	12.833	15.086
6	0.872	1.237	1.635	2.204	3.070	8..558	10.645	12.592	14.449	16.812
7	1.239	1.690	2.1675	2.833	3.822	9.803	12.017	14.067	16.013	18.475
8	1.646	2.180	2.733	3.490	4.594	11.030	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	5.380	12.242	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	6.179	13.442	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	6.989	14.631	17.275	19.675	21.920	24.725
12	3.571	4.404	5.226	6.304	7.807	15.812	18.549	21.026	23.337	26.217
13	4.107	5.009	5.892	7.042	8.634	16.985	19.812	22.362	24.736	27.688
14	4.660	5.629	6.571	7.790	9.467	18.151	21.064	23.685	26.119	29.141
15	5.229	6.262	7.261	8.547	10.307	19.311	22.307	24.996	27.488	30.578
16	5.812	6.908	7.962	9.312	11.152	20.465	23.542	26.296	28.845	32.000
17	6.408	7.564	8.672	10.085	12.002	21.615	24.769	27.587	30.191	33.409
18	7.015	8.231	9.390	10.865	12.857	22.760	25.989	28.869	31.526	34.805
19	7.633	8.907	10.117	11.651	13.716	23.900	27.204	30.144	32.852	36.191
20	8.260	9.591	10.851	12.443	14.578	25.038	28.412	31.410	34.170	37.566
21	8.897	10.283	11.591	13.240	15.445	26.171	29.615	32.671	35.479	38.932
22	9.542	10.982	12.338	14.041	16.314	27.301	30.813	33.924	36.781	40.289
23	10.196	11.6889	13.091	14.848	17.187	28.429	32.007	35.172	38.076	41.638
24	10.856	12.4015	13.848	15.658	18.062	29.553	33.196	36.415	39.364	42.980
25	11.524	13.120	14.611	16.473	18.940	30.675	34.382	37.652	40.647	44.314
26	12.198	13.844	15.379	17.292	19.820	31.795	35.563	38.885	41.923	45.642
27	12.879	14.573	16.151	18.114	20.703	32.912	36.741	40.113	43.195	46.963
28	13.565	15.308	16.928	18.939	21.588	34.027	37.916	41.337	44.461	48.278
29	14.256	16.047	17.708	19.768	22.475	35.139	39.087	42.557	45.722	49.588
30	14.953	16.791	18.493	20.599	23.364	36.250	40.256	43.773	46.979	50.892

Source: From Table IV of Fisher and Yates, *Statistical Tables for Biological, Agricultural and Medical Research*, Published by Longman Group Ltd (previously published by Oliver and Boyd, Edinburg, 1963).

Appendix : Table-5 Table of t

(One Tail Area)



Values of $t_{a,\,m}$

	Probability (Level of Significance)				
d.f (v)	**0.1**	**0.05**	**0.025**	**0.01**	**0.005**
1	3.078	6.3138	12.706	31.821	63.657
2	1.886	2.9200	4.3027	6.965	9.9248
3	1.638	2.3534	3.1825	4.541	5.8409
4	1.533	2.1318	2.7764	3.747	4.6041
5	1.476	2.0150	2.5706	3.365	4.0321
6	1.440	1.9432	2.4469	3.143	3.7074
7	1.415	1.8946	2.3646	2.998	3.4995
8	1.397	1.8595	2.3060	2.896	3.3554
9	1.383	1.8331	2.2622	2.821	3.2498
10	1.372	1.8125	2.2281	2.764	3.1693
11	1.363	1.7959	2.2010	2.718	3.1058
12	1.356	1.7823	2.1788	2.681	3.0545
13	1.350	1.7709	2.1604	2.650	3.0123
14	1.345	1.7613	2.1448	2.624	2.9768
15	1.341	1.7530	2.1315	2.602	2.9467
16	1.337	1.7459	2.1199	2.583	2.9208
17	1.333	1.7396	2.1098	2.567	2.8982
18	1.330	1.7341	2.1009	2.552	2.8784
19	1.328	1.7291	2.0930	2.539	2.8609
20	1.325	1.7247	2.0860	2.528	2.8453
21	1.323	1.7207	2.0796	2.518	2.8314
22	1.321	1.7171	2.0739	2.508	2.8188
23	1.319	1.7139	2.0687	2.500	2.8073
24	1.318	1.7109	2.0639	2.492	2.7969
25	1.316	1.7081	2.0595	2.485	2.7874
26	1.315	1.7056	2.0555	2.479	2.7787
27	1.314	1.7033	2.0518	2.473	2.7707
28	1.313	1.7011	2.0484	2.467	2.7633
29	1.311	1.6991	2.0452	2.462	2.7564
30	1.310	1.6973	2.0423	2.457	2.7500
35	1.3062	1.6896	2.0301	2.438	2.7239
40	1.3031	1.6839	2.0211	2.423	2.7045
45	1.3007	1.6794	2.0141	2.412	2.6896
50	1.2987	1.6759	2.0086	2.403	2.6778
60	1.2959	1.6707	2.0003	2.390	2.6603
70	1.2938	1.6669	1.994	2.381	2.6480
80	1.2922	1.6641	1.9945	2.374	2.6388
90	1.2910	1.6620	1.9901	2.364	2.6316
100	1.2901	1.6602	1.9867	2.364	2.6260
120	1.2887	1.6577	1.9840	2.358	2.6175
140	1.2876	1.6658	1.9799	2.353	2.6114
160	1.2869	1.6545	1.9771	2.350	2.6070
180	1.2863	1.6534	1.9749	2.347	2.6035
200	1.2858	1.6525	1.9733	2.345	2.6006
∞	1.282	1.645	1.96	2.326	2.576